

(91) $\sin x = \frac{2\sqrt{5}}{5}$ find $\csc x$.

$$\csc x = \frac{1}{\sin x}$$

$$\csc x = \frac{1}{\frac{2\sqrt{5}}{5}} \rightarrow (\text{give}).$$

$$= \frac{1}{\frac{2\sqrt{5}}{5}}$$

$$\csc x = \frac{5}{2\sqrt{5}}$$

(2) $\tan a = \frac{\sqrt{2}}{2}$ and $\sin a = -\frac{\sqrt{3}}{3}$ ($\cos a = ?$).

$$\cos a = \frac{\sin a}{\tan a}$$

$$= \frac{-\frac{\sqrt{3}}{3}}{\frac{\sqrt{2}}{2}}$$

$$= -\frac{\sqrt{3}}{3} \times \frac{2}{\sqrt{2}}$$

$$= \frac{-2\sqrt{3} \times \sqrt{2}}{3 \times \sqrt{2}}$$

$$\cos a = -\frac{2\sqrt{6}}{6}$$

$$\textcircled{3}) \quad \cot x = -\frac{\sqrt{3}}{2} \quad \text{and} \quad \cos x < 0.$$

$$\sin x = ?$$

$$\cot \theta = \frac{1}{\tan \theta}.$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\tan x = \frac{-2}{\sqrt{3}}.$$

$$\sin \theta = \tan \theta \cos \theta.$$

$$\begin{aligned} \sin x &= \frac{-2}{\sqrt{3}} \times \frac{\pi}{2} \\ &= \frac{-\pi}{\sqrt{3}}. \end{aligned}$$

$$\textcircled{4}) \quad \cos \theta = \frac{1}{7} \quad \sin \theta < 0; \quad \cot \theta = ?$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\therefore \cot \theta = \frac{\cos \theta}{\sin \theta}.$$

$$\cot \theta = \frac{1}{\frac{1}{7}} = \frac{7}{1}$$

$$\cot \theta = \frac{7}{1}$$

$$Q5) \cot\left(\frac{\pi}{2} - x\right) = -1.84 \quad \tan(x) ?$$

from Complementary Identities;

$$\tan \theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

$$\text{So, } \tan x = \cot\left(\frac{\pi}{2} - x\right)$$

$$\therefore \tan x = -1.84$$

from even-odd identities;

$$\tan(-x) = -\tan x$$

$$\frac{\tan x}{-1} = \frac{-1.84}{-1}$$

$$-\tan x = 1.84 = \tan(-x)$$

$$\tan(-x) = 1.84$$

$$\textcircled{6} \cdot \sin \theta = -0.57, \quad \cos\left(\theta - \frac{\pi}{2}\right) ?$$

$$\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\therefore \frac{\cos\left(\frac{\pi}{2} - \theta\right)}{-1} = \frac{-0.57}{-1}$$

$$\cos\left(\theta - \frac{\pi}{2}\right) = 0.57$$

$$\textcircled{7} \quad \sec x \cdot \sin^2 x + \cos x$$

$$= \frac{1}{\cos x} (1 - \cos^2 x) + \cos x$$

$$= \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} + \cos x$$

$$= \frac{1}{\cos x} - \cos x + \cos x$$

$$= \frac{1}{\cos x}$$

$$\textcircled{8} \quad \frac{\csc \beta - \sin \beta}{\sin \beta}$$

$$= \frac{\csc \beta}{\sin \beta} - \frac{\sin \beta}{\sin \beta}$$

$$= \frac{\csc \beta}{\sin \beta} - 1$$

$$\text{but } \frac{1}{\sin \beta} = \csc \beta$$

$$\therefore \csc^2 \beta - 1$$

$$= \csc^2 \beta - 1$$

$$\text{But } \csc^2 \beta = \cot^2 \beta + 1$$

$$= \cot^2 \beta + 1 - 1$$

$$= \cot^2 \beta$$

$$\text{Q9)} \quad \frac{\sin y + \sin y \cdot \cot^2 y}{\csc y}$$

$$= \frac{\sin y}{\csc y} + \frac{\sin y \cdot \cot^2 y}{\csc y}$$

$$= \sin y \cdot \sin y + \frac{\sin y (\csc^2 y - 1)}{\csc y}$$

$$= \sin^2 y + \frac{\sin y \csc^2 y}{\csc y} - \frac{\sin y}{\csc y}$$

$$= \frac{\sin y}{\csc y} - \frac{\sin y}{\csc y} + \frac{\sin y \csc^2 y}{\csc y}$$

$$= \sin y \csc y$$

$$= \sin y \times \frac{1}{\sin y}$$

$$= 1$$

$$\textcircled{10} (\sin \theta + 1) (\tan \theta - \sec \theta)$$

$$= (\sin \theta + 1) \left(\frac{\sin \theta}{\cos \theta} - \frac{1}{\cos \theta} \right)$$

$$= \frac{(\sin \theta + 1) (\sin \theta - 1)}{(\cos \theta)}$$

$$= \frac{\sin^2 \theta - 1}{\cos \theta}$$

$$= \frac{-\cos^2 \theta}{\cos \theta}$$

$$= -\cos \theta$$

$$(11) \frac{1 + \sec^2 x}{1 + \tan^2 x} = 1 + \cos^2 x$$

$$= \tan^2 \theta = -1 + \sec^2 \theta$$

$$\therefore \frac{1 + \sec^2 x}{1 - 1 + \sec^2 x}$$

$$= \frac{1 + \sec^2 x}{\sec^2 x}$$

$$= \frac{1}{\sec^2 x} + \frac{\sec^2 x}{\sec^2 x}$$

$$= \frac{1}{\sec^2 x} + 1$$

$$\text{But } (\cos x)^2 = \left(\frac{1}{\sec x}\right)^2$$

$$\text{hence } \cos^2 x = \frac{1}{\sec^2 x}$$

therefore, we can replace $\frac{1}{\sec^2 x}$ with $\cos^2 x$
to get $\cos^2 x + 1$

$$\text{hence } \frac{1 + \sec^2 x}{1 + \tan^2 x} = 1 + \cos^2 x \quad \square$$

(12)

$$\frac{\sin a}{1 - \cos a} - \cot a = \csc a$$

$$= \frac{\sin a}{1 - \cos a} - \frac{\cos a}{\sin a}$$

$$= \frac{\sin^2 a - \cos a (1 - \cos a)}{(1 - \cos a) (\sin a)}$$

$$= \frac{\sin^2 a - \cos a + \cos^2 a}{(1 - \cos a) (\sin a)}$$

$$\sin^2 a + \cos^2 a = 1$$

$$\therefore \frac{(1 - \cos a)}{(1 - \cos a) (\sin a)}$$

$$= \frac{1}{\sin a}$$

From reciprocal identities,

$$\csc a = \frac{1}{\sin a}$$

$$\therefore \frac{1}{\sin a} = \csc a \quad \square$$